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Finite-Temperature Corrections in the Dilated Chiral Quark Model

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ABSTRACT

We calculate the finite-temperature corrections in the dilated chiral quark model using the effective potential formalism. Assuming that the dilaton limit is applicable at some short length scale, we interpret the results to represent the behavior of hadrons in dense *and* hot matter. We obtain the scaling law, $\frac{f_\pi(T)}{f_\pi} = \frac{m_Q(T)}{m_Q} \simeq \frac{m_\sigma(T)}{m_\sigma}$ while we argue, using PCAC, that pion mass does not scale within the temperature range involved in our Lagrangian. It is found that the hadron masses and the pion decay constant drop faster with temperature in the dilated chiral quark model than in the conventional linear sigma model that does not take into account the QCD scale anomaly. We attribute the difference in scaling in heat bath to the effect of baryonic medium on thermal properties of the hadrons. Our finding would imply that the AGS

experiments (dense *and* hot matter) and the RHIC experiments (hot and dilute matter) will “see” different hadron properties in the hadronization phase.

One of the most challenging current topics in hadronic physics is to understand how physical properties of nuclear matter change as temperature and density are raised. The most exciting possibility is that under extreme conditions of temperature and density, the matter undergoes one or several phase transitions, such as meson condensations, chiral symmetry restoration and deconfinement transitions. Whereas several quark models predict the existence of various phase transitions, it is the lattice gauge calculations which provided an unambiguous evidence for the QCD phase transitions. While the most interesting physics would lie in the quark-gluon or Wigner-Weyl phase, experiments would necessarily involve hadronization and hence hadronic properties in the confined and/or Goldstone phase. It is therefore important to understand how the hadrons behave as the system approaches the critical point from below. For this purpose, the effective Lagrangian approach has proven to be quite useful and instructive. Indeed, effective chiral Lagrangians, constructed to describe the low-energy non-perturbative sector of QCD, have been used to predict the temperature dependence of pionic observables[1][2]. These studies including lattice calculations were, however, based on the hadronic vacuum of zero density. The question as to how the change of vacuum induced by density modifies hadron properties at finite temperature has not been addressed in a consistent manner.

A recent development in scaling effect in effective chiral Lagrangians[3] provides a possible means to address the question: How does dense medium affect the temperature dependence of hadronic properties?

To answer this question, we use the effective Lagrangian proposed by Beane and van Kolck[4] in the dilaton limit. These authors incorporate into a nonlinear σ model of constituent quarks and pions the effect of QCD scale anomaly and find that when the "mended symmetry" of Weinberg[5] is taken into account, the effective Lagrangian "linearizes" with the scalar field associated with the trace anomaly becoming a dilaton field σ. As discussed in [3], this Lagrangian describing the "mended symmetry" (dilaton) limit (which we shall refer to as "dilated chiral quark model") is suitable for physics of dense hadronic medium with density ρ and would naturally exhibit the density-dependent scaling proposed in [6]. There is, however, one caveat in this work as well as in work of ref.[3] that should be mentioned. The construction of the chiral Lagrangian at the dilaton limit leads naturally to a Lorentz-invariant description of the dynamics. In the presence of matter density, Lorentz invariance is broken, so the question is how the dilated Lagrangian should be

interpreted when applied to dense matter. We have no clear answer to this question except that in the dilaton limit, Lorentz-invariant Lagrangian appears to be a good first approximation. The remaining question then will be how to apply the dilaton limit to the presence of dense matter.

In this paper, we study the properties of hadrons at finite temperature using the Beane-van Kolck Lagrangian. To do this, we calculate the temperature-dependent effective potential in the dilated chiral quark model. We show first that the pion mass in the chiral limit remains zero at finite temperature as expected and observe scaling behaviors of the pion decay constant, f_π, the σ mass and the constituent-quark mass, m_Q, all reflecting the change with temperature of the vacuum expectation value of the dilaton field σ. In the Appendix, we reproduce the results of the one-loop effective potential by calculating explicitly the finite-temperature mass corrections in coupling-constant expansion.

The effective chiral quark Lagrangian supplemented with the QCD conformal anomaly that Beane and van Kolck[4] start with is

$$\begin{aligned} L &= \bar{\psi} i(\not{D} + \not{V})\psi + g_A \bar{\psi} \not{A}\gamma_5\psi - \sqrt{\kappa}\frac{m}{f_\pi}\bar{\psi}\psi\chi + \frac{1}{4}\kappa tr(\partial_\mu U \partial^\mu U^\dagger)\chi^2 \\ &+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}tr(G_{\mu\nu}G^{\mu\nu}) - V(\chi) + ... \end{aligned} \tag{1}$$

where $D_\mu = \partial_\mu + igG_\mu$, $V_\mu = \frac{1}{2}i(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$ and $A_\mu = \frac{1}{2}i(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$ with $\xi^2 = U = \exp(\frac{i2\pi_i T_i}{f_\pi})$. The scale anomaly of QCD appearing at quantum level[7] is contained in the potential V written as [8]

$$V(\chi) = -\frac{\kappa m_\chi^2}{8{f_\pi}^2}[\frac{1}{2}\chi^4 - \chi^4\ln(\frac{\kappa\chi^2}{f_\pi^2})], \tag{2}$$

where m_χ is the mass of the would-be dilaton field, χ. In the dilated chiral quark model, the dilaton field χ is replaced by an effective scalar field σ in the linear basis defined by $\sigma + i\vec{\tau}\cdot\vec{\pi} = U\chi\sqrt{\kappa}$. Near the mended symmetry regime which we assume is realized in dense medium, the dilaton limit ($m_\sigma \to 0$, $g_A \to 1$) is appropriate with both pions and σ making up a Goldstone quartet[4][9]. In the dilaton limit, the Lagrangian becomes

$$\begin{aligned} L &= i\bar{Q}\not{\partial}Q + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m}{f_\pi}\bar{Q}[\sigma - i\gamma_5\vec{\pi}\cdot\vec{\tau}]Q \\ &+ \frac{m_\sigma^2}{16f_\pi^2}(\sigma^2 + \vec{\pi}^2)^2 - \frac{m_\sigma^2}{8f_\pi^2}[(\sigma^2 + \vec{\pi}^2)^2\ln(\sigma^2 + \vec{\pi}^2)/f_\pi^2]. \end{aligned} \tag{3}$$

Since the minimum of the potential lies on the chiral circle, $\sigma^2 + \vec{\pi}^2 = f_\pi^2$ with $O(4)$ symmetry, we may choose

$$< \vec{\pi} >= \vec{\pi}_0 = 0 \quad < \sigma >= \sigma_0 = f_\pi, \tag{4}$$

with the pions remaining as Goldstone bosons. On this vacuum, the relevant excitations are

$$m_{\vec{\pi}}^2 = \frac{\partial^2 V}{\partial \vec{\pi}^2} \mid_{\sigma=\sigma_0, \vec{\pi}=0} = 0, \; m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \mid_{\sigma=\sigma_0, \vec{\pi}=0} = m_\sigma^2, \; m_Q = m. \tag{5}$$

We now introduce temperature. This can be done by means of the standard finite-temperature effective potential[10]. Define the background fields by a subscript c and shift the fields as $\sigma \to \sigma_c + \sigma$ and $\vec{\pi} \to \vec{\pi}_c + \vec{\pi}$ in eq.(3). The Lagrangian takes the form

$$\begin{aligned} L &= i\bar{Q} \not\partial Q - \frac{m}{f_\pi}\bar{Q}(\sigma_c - i\gamma_5 \vec{\pi}_c \cdot \vec{\tau})Q \\ &+ \frac{1}{2}\partial_\mu \vec{\pi}\partial^\mu \vec{\pi} + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}\vec{\pi} M_{\vec{\pi}}^2 \vec{\pi} - \frac{1}{2}\sigma M_\sigma^2 \sigma - \vec{\pi} M_{\sigma,\vec{\pi}}^2 \sigma \\ &+ \frac{m_\sigma^2}{16 f_\pi^2}(\sigma_c^2 + \vec{\pi}_c^2)^2 - \frac{m_\sigma^2}{8 f_\pi^2}(\sigma_c^2 + \vec{\pi}_c^2)^2 \ln[(\sigma_c^2 + \vec{\pi}_c^2)/f_\pi^2] \\ &+ \cdots \end{aligned} \tag{6}$$

where

$$\begin{aligned} M_{\vec{\pi}}^2 &= \frac{\partial^2 V}{\partial \vec{\pi}^2} \mid_{\sigma=\sigma_c, \vec{\pi}=\vec{\pi}_c} \\ &= \frac{m_\sigma^2}{2 f_\pi^2}[(\sigma_c^2 + \vec{\pi}_c^2)\ln((\sigma_c^2 + \vec{\pi}_c^2)/f_\pi^2) + 2\vec{\pi}_c^2 \ln((\sigma_c^2 + \vec{\pi}_c^2)/f_\pi^2) + 2\vec{\pi}_c^2] \\ M_\sigma^2 &= M_{\vec{\pi}}^2(\sigma_c \leftrightarrow \vec{\pi}_c) \\ M_{\sigma,\vec{\pi}}^2 &= \frac{m_\sigma^2}{f_\pi^2}\sigma_c \vec{\pi}_c(1 + \ln((\sigma_c^2 + \vec{\pi}_c^2)/f_\pi^2). \end{aligned} \tag{7}$$

The ellipsis in eq.(6) stands for the interactions of the fluctuating fields, $\vec{\pi}$ and σ, that are not relevant in calculating the effective potential at one-loop order. Some of them are explicitly shown in the Appendix.

Let us first calculate the contribution from the fermionic part of eq.(6). Using the inverse propagator,

$$\begin{aligned} iD^{-1} &= \not k - M_0 \\ M_0 &= \frac{m}{f_\pi}(\sigma_c - i\gamma_5 \vec{\pi}_c \cdot \vec{\tau}), \end{aligned} \tag{8}$$

the contribution to the one-loop finite-temperature effective potential[10] can be written as

$$
\begin{aligned}
V_Q^\beta &= i\int\frac{d^4k}{(2\pi)^4}\ln\det(\not k - M_0) \\
&= 2i\int\frac{d^4k}{(2\pi)^4}\ln\det(k^2-M^2) \\
&= -4\frac{1}{2\pi^2\beta^4}\int_0^\infty dx\, x^2\sum_i \ln(1+e^{-(x^2+\beta^2M_i^2)^{1/2}})
\end{aligned} \tag{9}
$$

where $M^2=\frac{m^2}{f_\pi^2}(\sigma_c^2+\vec{\pi}_c^2)$ and $\beta=\frac{1}{kT}$. The first determinant involves Dirac and isospin indices of the constituent quark, and the second one does only isospin index. The sum over i goes over the eigenvalues M_i of the matrix M. For small $\beta^2M_i^2$ [1]

$$
\begin{aligned}
V_Q^\beta &\sim \sum_i\left[\frac{7\pi^2}{180\beta^4}+\frac{1}{12\beta^2}\frac{m^2}{f_\pi^2}(\sigma_c^2+\vec{\pi}_c^2)\right] \\
&= \frac{7\pi^2}{90\beta^4}+\frac{1}{6\beta^2}\frac{m^2}{f_\pi^2}(\sigma_c^2+\vec{\pi}_c^2).
\end{aligned} \tag{10}
$$

Since we are interested in the vacuum structure that depends only on σ_c or $\vec{\pi}_c$, we shall hereafter retain only those terms in the effective potential that depend upon them and drop the terms that do not. Then we get

$$
V_Q^\beta \sim \frac{1}{6\beta^2}\frac{m^2}{f_\pi^2}(\sigma_c^2+\vec{\pi}_c^2). \tag{11}
$$

Next we compute the contribution from the meson loops. Using eq.(7), the one-loop finite-temperature contributions from σ and $\vec{\pi}$ can also be readily calculated[10]:

$$
\begin{aligned}
V^\beta(\vec{\pi}_c,\sigma_c) = &- \frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\,\det(k^2-M_\sigma^2) \\
&- \frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\,\det\left(k^2-M_\pi^2+\frac{(M_{\sigma,\vec{\pi}}^2)^2}{k^2-M_\sigma^2}\right)
\end{aligned}
$$

[1] Since the masses of the particles involved in this case are assumed to be small in the dilaton limit, the expansion can be used even at quite low temperatures as long as the temperature is higher than the masses.

$$
\begin{aligned}
&= - \frac{i\hbar}{2}\int \frac{d^4k}{(2\pi)^4}\ln\left((k^2)^2-(M_\sigma^2+M_{\vec{\pi}}^2)k^2+(M_{\sigma,\vec{\pi}}^2)^2\right)\\
&= + \frac{1}{2\pi^2\beta^4}\int_0^\infty x^2dx[\ln(1-e^{-(x^2+\beta^2R_1^2)^{1/2}})\\
&+ \ln(1-e^{-(x^2+\beta^2R_2^2)^{1/2}})] \qquad (12)
\end{aligned}
$$

where R_1^2 and R_2^2 are the roots of the quadratic equation in k^2,

$$
(k^2)^2-(M_\sigma^2+M_{\vec{\pi}}^2)k^2+(M_{\sigma,\vec{\pi}}^2)^2=0.
$$

For small $\beta^2R_i^2$,

$$
V^\beta(\vec{\pi}_c,\sigma_c)\simeq\frac{1}{24\beta^2}(R_1^2+R_2^2)=\frac{1}{24\beta^2}(M_\sigma^2+M_{\vec{\pi}}^2). \qquad (13)
$$

Summing up, we have the full one-loop temperature-dependent effective potential

$$
\begin{aligned}
V_{eff} &= V_0+V_Q^\beta+V^\beta(\vec{\pi},\sigma)\\
&= -\frac{m_\sigma^2}{16f_\pi^2}(\sigma^2+\vec{\pi}^2)^2+\frac{m_\sigma^2}{8f_\pi^2}(\sigma^2+\vec{\pi}^2)^2\ln[\frac{(\sigma^2+\vec{\pi}^2)}{f_\pi^2}]\\
&+ \frac{1}{6\beta^2}\frac{m^2}{f_\pi^2}(\sigma^2+\vec{\pi}^2)+\frac{1}{24\beta^2}(M_\sigma^2+M_{\vec{\pi}}^2) \qquad (14)
\end{aligned}
$$

where we have omitted the subscript c in the σ and π fields: it should be understood that they are classical background fields. Now since chiral symmetry must be intact at finite temperature, we shall assume that the minimum of the effective potential, eq.(14), lies on the chiral circle, $\sigma^2+\vec{\pi}^2=$ const., as in eq.(3). Hence in the chiral limit, pions will remain as Goldstone bosons, with $<\vec{\pi}>=0$ and $<\sigma>=\sigma_0\neq 0$. Defining the dimensionless quantities

$$
x^2=\frac{\vec{\pi}^2}{f_\pi^2},\ y^2=\frac{\sigma^2}{f_\pi^2},\ r^2=\frac{m^2}{m_\sigma^2},\ t=\frac{1}{\beta f_\pi}, \qquad (15)
$$

we can rewrite the effective potential as

$$
\begin{aligned}
V_{eff} &= \frac{m_\sigma^2f_\pi^2}{16}[-(x^2+y^2)^2+2(x^2+y^2)^2\ln(x^2+y^2)\\
&+ \frac{4}{3}t^2(x^2+y^2)\ln(x^2+y^2)+\frac{2}{3}t^2(x^2+y^2)+\frac{8}{3}t^2r^2(x^2+y^2)]. \qquad (16)
\end{aligned}
$$

Now we define the temperature-dependent vacuum $\sigma_0(T)$, by extremizing the effective potential as

$$\frac{\partial V_{eff}}{\partial \sigma} |_{\sigma=\sigma_0(T),\vec{\pi}=0} = 0, \quad \frac{\partial V_{eff}}{\partial \vec{\pi}} |_{\sigma=\sigma_0(T),\vec{\pi}=0} = 0, \tag{17}$$

and obtain

$$8y_0^2 \ln y_0^2 + \frac{8}{3}t^2 \ln y_0^2 + 4t^2 + \frac{16}{3}t^2 r^2 = 0, \tag{18}$$

where $y_0^2 = \sigma_0(T)^2/f_\pi^2$. Below y will correspond to y_0. Equation (18) can be solved to get the temperature-dependent vacuum expectation value of the σ field. One can readily check from eq.(18) that in the absence of temperature, $\sigma_0 = f_\pi$ $(y = 1)$. One can also verify that the condition (18) automatically guarantees that the pion mass remains zero at all temperatures as required by chiral invariance:

$$\frac{\partial^2 V_{eff}}{\partial \vec{\pi}^2} |_{\sigma=\sigma_0(T),\vec{\pi}=0} = \frac{m_\sigma^2}{16}[8y^2 \ln y^2 + \frac{8}{3}t^2 \ln y^2 + 4t^2 + \frac{16}{3}t^2 r^2] = 0. \tag{19}$$

PCAC implies that even when chiral symmetry is broken by the current quark masses of the u and d quarks, the pion mass will remain small and unmodified by temperature.

For small t, we can approximate

$$y = 1 + \delta, \quad \sigma_0(T) = f_\pi(1 + \delta). \tag{20}$$

From eqs.(18) and (20), it follows that

$$\delta = -\frac{1}{4}t^2, \tag{21}$$

for $r^2 \ll 1$. Now identifying $\sigma_0(T)$ with $f_\pi(T)$ and using eq.(21), we obtain the scaling of the pion decay constant

$$\frac{f_\pi(T)}{f_\pi} = 1 + \delta = 1 - \frac{T^2}{4f_\pi^2} \tag{22}$$

Next, the σ mass is calculated from the effective potential:

$$\begin{aligned} m_\sigma^2(T) &= \frac{1}{f_\pi^2}\frac{\partial^2 V_{eff}}{\partial y^2} |_{\sigma=\sigma_0(T),\vec{\pi}=0} \\ &= \frac{m_\sigma^2}{16}[24y^2 \ln y^2 + 16y^2 + \frac{8}{3}t^2 \ln y^2 + \frac{28}{3}t^2]. \end{aligned} \tag{23}$$

Using eq.(20) and (21), we have

$$m_\sigma^2(T) = m_\sigma^2[y^2 + y^2 lny^2 + \frac{1}{3}t^2] \simeq m_\sigma^2(1 + \frac{8}{3}\delta), \tag{24}$$

and hence

$$\frac{m_\sigma(T)}{m_\sigma} \simeq 1 + \frac{4}{3}\delta = 1 - \frac{1}{3}\frac{T^2}{f_\pi^2} \tag{25}$$

The temperature-dependent constituent quark mass can be read off directly from eq.(3):

$$m_Q(T) = \frac{m}{f_\pi}\sigma_0(T), \tag{26}$$

or

$$\frac{m_Q(T)}{m_Q} = 1 + \delta = 1 - \frac{T^2}{4f_\pi^2}. \tag{27}$$

We thus find that $m_Q(T)$, $f_\pi(T)$ and $m_\sigma(T)$ satisfy approximately the same scaling in temperature as in density postulated in [6].

It should be noted that these results are, however, different from those of σ models in matter-free space, namely the results of chiral perturbation theory at one loop[1], in two aspects. First, the temperature corrections are much bigger for two reasons: the coefficient multiplying T^2/f_π^2 in eq.(22) is three times bigger than in chiral perturbation calculation and furthermore $f_\pi(\rho)$ in medium is smaller than f_π in matter-free space[6] and secondly, the $\sigma(T)$ scales slightly faster than $f_\pi(T)$ in contrast to what we expect from the linear σ model as we shall show explicitly below. We can trace the differences to the presence of the logarithmic-type potential introduced by the QCD scale anomaly in the dilated chiral model.

In the linear sigma model[13], the Lagrangian is

$$L = \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{8}\frac{m_\sigma^2}{f_\pi^2}[\sigma^2 + \vec{\pi}^2 - f_\pi^2]^2, \tag{28}$$

where λ and μ in ref. [13] are replaced by f_π and m_σ, $f_\pi = (\mu^2/\lambda)^{1/2}$, $m_\sigma^2 = 2\mu^2$. The finite-temperature effective potential up to one loop is

$$V_{eff} = -\frac{1}{4}m_\sigma^2(\sigma^2+\vec{\pi}^2)+\frac{1}{8}\frac{m_\sigma^2}{f_\pi^2}(\sigma^2+\vec{\pi}^2)^2 + \frac{T^2}{24}[\frac{m_\sigma^2}{f_\pi^2}(\sigma^2+\vec{\pi}^2)-\frac{1}{2}m_\sigma^2], \tag{29}$$

or using the dimensionless variables,

$$V_{eff} = \frac{1}{4}f_\pi^2 m_\sigma^2[-(x^2+y^2)+\frac{1}{2}(x^2+y^2)^2+\frac{t^2}{6}(x^2+y^2-\frac{1}{2})]. \tag{30}$$

The vacuum condition, $\frac{\partial V_{tot}^\beta}{\partial y} = 0$ with $x = 0$, leads to

$$y^2 - 1 + \frac{t^2}{6} = 0. \tag{31}$$

In the low-temperature approximation, eq.(20), we get

$$\delta(T) \simeq -\frac{T^2}{12f_\pi^2} \tag{32}$$

and the temperature correction to f_π:

$$f_\pi(T) = f_\pi y = f_\pi(1-\frac{T^2}{12f_\pi^2}). \tag{33}$$

We see that the temperature correction in eq.(33) is smaller than in eq.(22). This result is also obtained in chiral perturbation theory[1] (with the non-linear σ model). The temperature dependence of the σ mass in the linear σ model is

$$m_\sigma^2(T) = -\frac{1}{2}m_\sigma^2+\frac{3}{2}m_\sigma^2 y^2+\frac{T^2}{12}\frac{m_\sigma^2}{f_\pi^2} \tag{34}$$

which with eq(31) can be reduced to

$$m_\sigma(T) = m_\sigma\, y = m_\sigma(1-\frac{T^2}{12f_\pi^2}) \tag{35}$$

where eq.(32) has been used for the last equality. One can see that the σ mass is essentially proportional to y or $\sigma_0(T)$ as for $f_\pi(T)$, eq.(35). This explains why in the linear sigma model, the "universal scaling"

$$\frac{f_\pi(T)}{f_\pi} = \frac{m_\sigma(T)}{m_\sigma} \tag{36}$$

holds. In contrast, in the dilated chiral quark model, while $f_\pi(T)$ is directly proportional to y as it is identified with $\sigma_0(T)$, $m_\sigma(T)$ is not simply related to y, see eq.(24). This is due to the logarithmic term in the effective potential.

In summary, we have calculated the one-loop effective potential at finite temperature to determine the temperature dependence of the light hadron masses (pion, σ and constituent quark) and the pion decay constant, using the dilated chiral quark model in the dilaton limit. Since in the dilaton limit, the σ mass is small, the low-temperature expansion we are making is justified. We found that the BR scaling postulated in dense matter also holds approximately in hot matter in the dilaton limit:

$$\frac{f_\pi(T)}{f_\pi} = \frac{m_Q(T)}{m_Q} \simeq \frac{m_\sigma(T)}{m_\sigma}. \tag{37}$$

As stated before, the question remains as to how relevant the dilaton limit on which the Beane-van Kolck construction relies is to dense nuclear matter. In the dilaton limit where Weinberg's mended symmetry applies, the effective Lagrangian preserves Lorentz invariance[2]. But the presence of the Fermi sea in dense matter breaks Lorentz invariance, so the valid question is how good is the assumption that dense matter can be described by the dilaton limit. If, however, we assume that the dilated chiral quark model describes "vacuum properties" in dense hadronic medium as argued in [3], the more rapid decrease of f_π, m_σ and m_Q with temperature compared with free-space σ models would imply that heavy-ion experiments performed in hot and dense medium probe different hadron properties than in hot but dilute medium. This prediction could be checked on lattice as well as in heavy-ion experiments.

[2]For instance, in medium, one expects that the pion decay constant f_π (and also the axial coupling constant g_A etc.) has two components, one for the time component of the axial current, say, $f_\pi^{(t)}$ and another for the space component, $f_\pi^{(s)}$. In general, these two are not the same. However in the dilaton limit, one recovers $f_\pi^{(t)} = f_\pi^{(s)}$.

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Appendix

In this Appendix, we explicitly calculate the finite-temperature corrections to the σ mass in a coupling-constant expansion up to order $\frac{m_\sigma^2}{f_\pi^2}$ to check the results obtained from the one-loop effective potential. After shifting the fields as $\sigma \to f_\pi + \sigma$ and $\vec{\pi} \to \vec{\pi}$ in eq.(3), we get the Lagrangian density:

$$\begin{aligned} L &= \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m}{f_\pi}\bar{Q}[\sigma - i\gamma_5\vec{\pi}\cdot\vec{\tau}]Q + \frac{m_\sigma^2}{16f_\pi^2}((f_\pi+\sigma)^2 \\ &+ \vec{\pi}^2)^2 - \frac{m_\sigma^2}{8f_\pi^2}((f_\pi+\sigma)^2+\vec{\pi}^2)^2\ln[1+(\sigma^2+2f_\pi\sigma+\vec{\pi}^2)/f_\pi^2]. \end{aligned} \tag{1}$$

The pion-σ interaction vertices can be obtained by expanding the potential terms in eq.(1),

$$\begin{aligned} L &\simeq i\bar{Q}\,\partial\!\!\!/ Q - m\bar{Q}Q + \frac{1}{2}\partial_\mu\vec{\pi}\partial^\mu\vec{\pi} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma\sigma^2 \\ &- \frac{m}{f_\pi}\bar{Q}[\sigma - i\gamma_5\vec{\pi}\cdot\vec{\tau}]Q - \frac{1}{2}\frac{m_\sigma^2}{f_\pi}\sigma\vec{\pi}^2 - \frac{5}{6}\frac{m_\sigma^2}{f_\pi}\sigma^3 \\ &- \frac{3}{4}\frac{m_\sigma^2}{f_\pi^2}\sigma^2\vec{\pi}^2 - \frac{1}{8}\frac{m_\sigma^2}{f_\pi^2}\vec{\pi}^4 - \frac{11}{24}\frac{m_\sigma^2}{f_\pi^2}\sigma^4 + \cdots, \end{aligned} \tag{2}$$

where only the relevant interactions for finite-temperature corrections up to order $\lambda = \frac{m_\sigma^2}{f_\pi^2}$ are shown explicitly.

The observation that the $\vec{\pi}$ field remains massless with the one-loop finite temperature effective potential can be verified by checking that there is no temperature-dependent corrections up to order of λ. The relevant diagrams are depicted in Fig.1. One can show that Fig.1(a)-(e) cancel among themselves and Fig.1(f) and (g) cancel each other exactly.

The relevant diagrams for temperature corrections to the σ mass are shown in Fig. 2. They give corrections to order λ. A simple calculation gives

$$\Sigma_{tot}(0) = \Sigma_a(0) + \Sigma_b(0) + \Sigma_c(0) + \Sigma_f(0) = -\frac{2}{3}\lambda T^2. \tag{3}$$

There are no mass corrections from Fig. 2 (d) and (e) [11][12]. Hence we get

$$m_\sigma^2(T) = m_\sigma^2 - \frac{2}{3}\lambda T^2 \tag{4}$$

which is exactly eq.(24).

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